

- 3.4** Consider the reel game described in Problem 3.3.
- What is the probability of getting a diamond on every reel?
 - What is the probability of getting a heart on exactly three of the reels?
 - What is the probability of getting a heart on at least three reels?
 - The middle reel on one machine always sticks, displaying a diamond. Calculate the probability of getting three diamonds on this machine.

3.5 Binomial expressions such as $(x + y)^3$ can be expanded using the distributive property. For example, the expanded forms of several binomial expressions are shown below:

$$(x + y)^0 = 1$$

$$(x + y)^1 = x + y$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

- Examine the expanded form of each of the binomial expressions above. Describe the sum of the exponents on each term in the expanded form with respect to the exponent of the original expression.
- For each expression, describe the relationship between the coefficients of each term and Pascal's triangle.
- Rewrite the expanded form of $(x + y)^4$ using the notation for combinations, $C(n, r)$.
- Use your results in Parts **a–c** to suggest an expanded form for the general binomial $(x + y)^n$.

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- 3.6** Experimental data shows that when a thumbtack is tossed in the air, it will land point up 75% of the time, and point down 25% of the time.
- Does tossing a thumbtack in the air and observing the outcome represent a binomial experiment?
 - What is the probability that when a tack is tossed 10 times, it lands point down exactly 6 times?
- 3.7** A married couple plans to have four children. Assuming that the probability that each child is a girl is 0.5, what is the probability that their four children will include:
- 3 boys and 1 girl?
 - 4 girls?
 - 2 boys and 2 girls?

- 3.8** Consider a test which contains 10 multiple-choice questions. Each question offers 4 possible choices for the answer, but only one of them is correct. To pass this test, students must obtain at least 6 correct answers. If a student guesses the answer to each question at random, what is the probability of passing the test?
- 3.9** A certain airplane has two engines. The probability that any one engine will fail during a transcontinental flight is 0.001. Assuming that the event of one engine failing is independent of the other engine failing, determine the probability of each of the following.
- a transcontinental flight will be completed without engine failure
 - both engines will fail
 - at least one engine will fail.
- 3.10** On January 28, 1986, the space shuttle *Challenger* exploded shortly after launch. The cause of this tragedy was traced to the failure of 1 of the 6 sealed joints on the booster rockets. Assuming that each joint has a 0.977 success rate, and that the failure of any one joint is independent of the failure of any of the others, calculate the probability that at least 1 of the 6 joints fails.
- 3.11** According to the **binomial theorem**, the expansion of $(x + y)^n$, where n is a whole number, is the sum of $(n + 1)$ terms, as follows:
- $$(x + y)^n = C(n, n)x^n + C(n, n - 1)x^{n-1}y + C(n, n - 2)x^{n-2}y^2 + \dots + C(n, 0)y^n$$
- Use the binomial theorem to expand each of the binomials below.
- $(x - y)^6$
 - $(x^2 + 3y)^3$
 - $(2xy - 5)^4$

Activity 4

The video gaming machine industry is highly competitive. Besides designing games that keep players interested and entertained, High Tech Games must also ensure that its products generate a profit.

Whenever someone uses a gaming machine, there are costs involved. For the player, the cost is the price required to play. For the machine's owners, the costs include paying off the credits earned by successful players.