5 MINUTE CHECK

Perform the indicated operation, and write the result in the form $a+bi$.

1. $(2+3i)+(-1+5i)$

2. $(-3+2i)(3-4i)$

Factor the quadratic equation.

3. $2x^2-9x-5$

Solve the quadratic equation.

4. $x^2-6x+10=0$

List all potential rational zeros.

5. $4x^4-3x^2+x+2$

Fundamental Theorem of Algebra

A polynomial function of degree n has n complex zeros (real and nonreal). Some of these zeros may be repeated.

If $f(x)$ is a polynomial function of degree $n > 0$, then $f(x)$ has precisely n linear factors and $f(x) = a(x - z_1)(x - z_2)\ldots(x - z_n)$ where a is the leading coefficient of $f(x)$ and $z_1, z_2, \ldots, z_n$ are the complex zeros of $f(x)$. The z_i are not necessarily distinct numbers; some may be repeated.

The following statements about a polynomial function f are equivalent if k is a complex number:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$
2. k is a zero of the function f.
3. $x - k$ is a factor of $f(x)$.

Write the polynomial function in standard form, identify the zeros of the function and the x-intercepts of its graph.

$$f(x) = (x - 3i)(x + 3i) = x^2 - (3i)^2 = x^2 - 9i^2$$

$$= x^2 + 9$$

$(\pm 3i, 0)$ $\quad x = \pm 3i$

Suppose that $f(x)$ is a polynomial function with real coefficients. If a and b are real numbers with $b \neq 0$ and $a + bi$ is a zero of $f(x)$, then its complex conjugate $a - bi$ is also a zero of $f(x)$.

Write a polynomial of minimum degree in standard form with real coefficients whose zeros include 2, -3, and $1+i$. $1-i$

$F(x) = (x-2)(x+3)(x-(1+i))(x-(1-i))$

$= (x^2+x-6)(x^2-2x+2) = x^4 - x^3 - 6x^2 + 14x - 12$

$x - 1 - i$
$x - 1 + i$
$x^2 - x - xi$
$-x + xi + 1 + i$
$+i$
$-i^2$

$x^2 - 2x + 1 - (-1)$
$x^2 - 2x + 2$

$x^2 + x - 6$
$x^2 - 2x + 2$
$x^4 + x^3 - 6x^2$
$-2x^3 - 2x^2 + 12x$
$2x^2 + 2x - 12$

$x^4 - x^3 - 6x^2 + 14x - 12$

Every polynomial function with real coefficients can be written as a product of linear factors and irreducible quadratic factors, each with real coefficients.

$$(x-2)(x^2+4) = f(x)$$

Special Assignment Example 2.5

Write $f(x) = 3x^5 + x^4 - 24x^3 - 8x^2 - 27x - 9$ as a product of linear and irreducible quadratic factors, each with real coefficients.