Answers for Lesson 6-1 Exercises

1. $10x + 5$; linear binomial
2. $-3x + 5$; linear binomial
3. $2m^2 + 7m - 3$; quadratic trinomial
4. $x^4 - x^3 + x$; quartic trinomial
5. $2p^2 - p$; quadratic binomial
6. $3a^3 + 5a^2 + 1$; cubic trinomial
7. $-x^5$; quintic monomial
8. $12x^4 + 3$; quartic binomial
9. $5x^3$; cubic monomial
10. $-2x^3$; cubic monomial
11. $5x^2 + 4x + 8$; quadratic trinomial
12. $-x^4 + 3x^3$; quartic binomial
13. $y = x^3 + 1$
14. $y = 2x^3 - 12$
15. $y = 1.5x^3 + x^2 - 2x + 1$
16. $y = -3x^3 - 10x^2 + 100$
17. **a.** males: $y = -0.003357x^2 + 0.3253x + 67.05$
    females: $y = -0.001929x^2 + 0.2321x + 74.89$

    b. males: $y = -0.0001000x^3 + 0.002643x^2 + 0.2393x + 67.17$
    females: $y = 0.0001667x^3 - 0.001193x^2 + 0.3755x + 74.69$

    c. For males, the models offer similar fit. For females, the cubic model is a better fit.
18. $y = x^3 - 2x^2$; 4335
19. $y = x^3 - 10x^2$; 2023
20. $y = -0.5x^3 + 10x^2$; 433.5

21. $y = -0.03948x^3 + 2.069x^2 - 17.93x + 106.9$; 206.07

22. $y = -0.007990x^3 + 0.4297x^2 - 6.009x + 43.57$; 26.34

23. $y = 0.01002x^3 - 0.3841x^2 + 5.002x + 2.132$; 25.39

24. Check students' work.

25. $x^3 + 4x$; cubic binomial

26. $-4a^4 + a^3 + a^2$; quartic trinomial

27. 7; constant monomial

28. $6x^2$; quadratic monomial

29. $x^4 + 2x^3$; quartic binomial

30. $\frac{1}{2}x^5 + \frac{2}{3}x$; quintic binomial

31. **a.** $V = 10\pi r^2$

b. $V = \frac{2}{3}\pi r^3$

c. $V = \frac{2}{3}\pi r^3 + 10\pi r^2$

32. Answers may vary. Sample: Cubic functions represent curvature in the data. Because of their turning points they can be unreliable for extrapolation.

33. $-c^2 + 16$; binomial

34. $-9d^3 - 13$; binomial

35. $16x^2 - x - 5$; trinomial

36. $2x^3 - 6x + 17$; trinomial

37. $a + 4b$; binomial

38. $-12y$; monomial

39. $8x^2 - 6y$; binomial

40. $-3a + 2$; binomial

41. $2x^3 + 9x^2 + 5x + 27$; polynomial of 4 terms

42. $-4x^4 - 3x^3 + 5x - 54$; polynomial of 4 terms

43. $80x^3 - 109x^2 + 7x - 75$; polynomial of 4 terms

44. $2x^3 - 2x^2 + 8x - 27$; polynomial of 4 terms

45. $6a^2 + 3ab - 8$; trinomial

46. $8x^3 + 2x^2$; binomial

47. $30x^3 - 10x^2$; binomial

48. $2a^3 - 5a^2 - 2a + 5$; polynomial of 4 terms

49. $b^3 - 6b^2 + 9b$; trinomial

50. $x^3 - 6x^2 + 12x - 8$; polynomial of 4 terms

51. $x^4 + 2x^2 + 1$; trinomial

52. $8x^3 + 60x^2 + 150x + 126$; polynomial of 4 terms

53. $a^3 - a^2b - b^2a + b^3$; polynomial of 4 terms

54. $a^4 - 4a^3 + 6a^2 - 4a + 1$; polynomial of 5 terms

55. $12s^3 + 61s^2 + 68s - 21$; polynomial of 4 terms

56. $x^3 + 2x^2 - x - 2$; trinomial

57. $8c^3 - 26c + 12$; trinomial

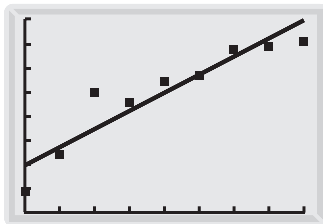
58. $s^4 - 2t^2s^2 + t^4$; trinomial

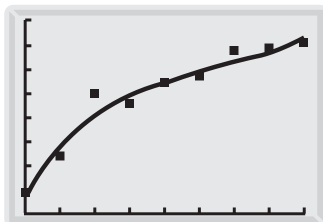
59. a. $y = 0.6877x + 47.97$

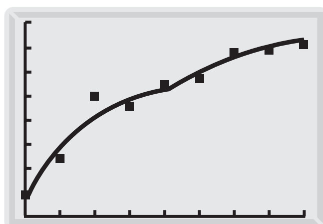
$y = 0.0007125x^3 - 0.06366x^2 + 2.1690x + 41.5929$

$y = -0.00005632x^4 + 0.005218x^3 - 0.1757x^2 + 3.0459x + 40.7482$

b.







Answers may vary. Sample: The quartic model fits best.

c. For sample in part (b), 71.68×10^{15} Btu

60. 2.5×10^8 cm^3

61. a. up 4 units

b. $y = 4x^3$ is more narrow.

c. $y = x^3$

Answers for Lesson 6-2 Exercises

1. $x^2 + x - 6$

2. $x^3 + 12x^2 + 47x + 60$

3. $x^3 - 7x^2 + 15x - 9$

4. $x^3 + 4x^2 + 4x$

5. $x^3 + 10x^2 + 25x$

6. $x^3 - x$

7. $x(x - 6)(x + 6)$

8. $3x(3x - 1)(x + 1)$

9. $5x(2x^2 - 2x + 3)$

10. $x(x + 5)(x + 2)$

11. $x(x + 4)^2$

12. $x(x - 9)(x + 2)$

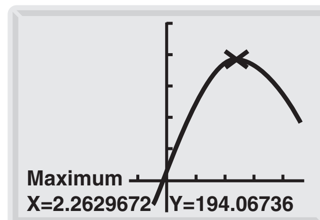
13. about 24.2, −1.4, 0, −5, 1

14. about 5.0, −16.9, 2, 6, 8

15. **a.** $h = x$, $\ell = 16 - 2x$, $w = 12 - 2x$

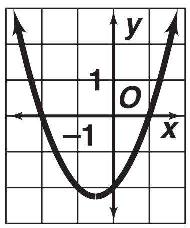
b. $V = x(16 - 2x)(12 - 2x)$

c.

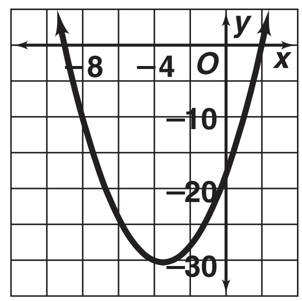


194 in.^{3}, 2.26 in.

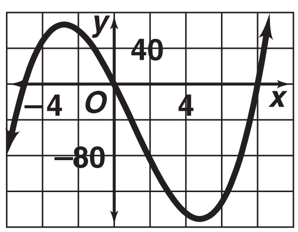
16. 1, −2



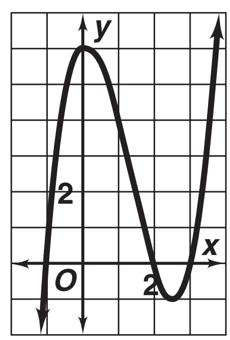
17. 2, −9



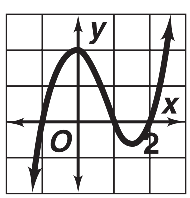
18. 0, −5, 8



19. −1, 2, 3



20. −1, 1, 2



21. $y = x^3 - 18x^2 + 107x - 210$

22. $y = x^3 + x^2 - 2x$

23. $y = x^3 + 9x^2 + 15x - 25$

24. $y = x^3 - 9x^2 + 27x - 27$

25. $y = x^3 + 2x^2 - x - 2$

26. $y = x^3 + 6x^2 + 11x + 6$

27. $y = x^3 - 2x^2$

28. $y = x^3 - \frac{7}{2}x^2 - 2x$

29. -3 (mult. 3)

30. 0, 1 (mult. 3)

31. $-1, 0, \frac{1}{2}$

32. $-1, 0, 1$

33. 4 (mult. 2)

34. 1, 2 (mult. 2)

35. $-\frac{3}{2}$, 1 (mult. 2)

36. -1 (mult. 2), 1, 2

37. 2 x^3 blocks, 15 x^2 blocks, 31 x blocks, 12 unit blocks

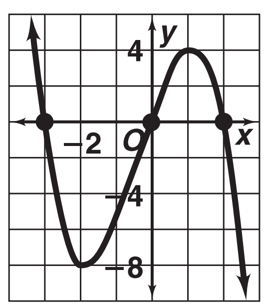
38. **a.** $V = 2x^3 + 15x^2 + 31x + 12$; $2x^3 + 7x^2 + 7x + 2$

b. $V = 8x^2 + 24x + 10$

39. $V = 12x^3 - 27x$

40. **a.** $h = x + 3$; $w = x$

b.



0, −3, 2; where the volume is zero

c. $0 < x < 2$

d. about 4.06 ft^3

41. $y = -2x^3 + 9x^2 - x - 12$

42. $y = 5x^4 - 23x^3 - 250x^2 + 1164x + 504$

43. $y = 3x(x - 8)(x - 1)$ **44.** $y = -2x(x + 5)(x - 4)$

45. $y = x^2(x + 4)(x - 1)$ **46.** $y = \frac{1}{2}x\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$

47. about 10.5, −7.1; $\frac{3}{2}$, 4, 6

48. about 0.9, −6.9, −1.4; 0, −3, −1, 1

49. about −2.98, −6.17; 1.5 **50.** none, −1; −2, 0

51–53. Answers may vary. Samples are given.

51. $y = x^3 - 3x^2 - 10x$

52. $y = x^3 - 21x^2 + 147x - 343$

53. $y = x^4 - 4x^3 - 7x^2 + 22x + 24$

54. −4, 5 (mult. 3) **55.** 0 (mult. 2), −1 (mult. 2) **56.** 0, 6, −6

57. Answers may vary. Sample: Write the polynomial in standard form. The constant term is the value of the y-intercept.

58. 1 ft

59. Answers may vary. Sample: $y = x^4 - x^2$, and zeros are 0, ±1.

60. Answers may vary. Sample: The linear factors can be determined by examining the x-intercepts of the graph.

61. $x + 2a$

62. **a.** $A = -x^3 + 2x^2 + 4x$
b. $6\frac{7}{8}$ square units

63. Answers may vary. Sample: $y = (x - 1)(x + 1)(x - i)(x + i)$; $y = x^4 - 1$

64. **a.** Answers may vary. Sample: translation to the right 4 units
b. No; the second graph is not the result of a horizontal translation.
c. Answers may vary. Sample: rotation of 180° about the origin

Answers for Lesson 6-3 Exercises

1. $x - 8$

2. $3x - 5$

3. $x^2 + 4x + 3$, R 5

4. $2x^2 + 5x + 2$

5. $3x^2 - 7x + 2$

6. $9x - 12$, R -32

7. $x - 10$, R 40

8. $x^2 + 4x + 3$

9. no

10. yes

11. yes

12. no

13. $x^2 + 4x + 3$

14. $x^2 - 2x + 2$

15. $x^2 - 11x + 37$, R -128

16. $x^2 + 2x + 5$

17. $x^2 - x - 6$

18. $-2x^2 + 9x - 19$, R 40

19. $x + 1$, R 4

20. $3x^2 + 8x - 3$

21. $x^2 - 3x + 9$

22. $6x - 2$, R -4

23. $y = (x + 1)(x + 3)(x - 2)$

24. $y = (x + 3)(x - 4)(x - 3)$

25. $\ell = x + 3$ and $h = x$

26. 18

27. 0

28. 0

29. 12

30. 168

31. 10

32. 51

33. 0

34. $P(a) = 0$; $x - a$ is a factor of $P(x)$.

35. $x - 1$ is not a factor of $x^3 - x^2 - 2x$ because it does not divide into $x^3 - x^2 - 2x$ evenly.

36. Answers may vary. Sample: $(x^2 + x - 4) \div (x - 2)$

37. $x^2 + 4x + 5$

38. $x^3 - 3x^2 + 12x - 35$, R 109

39. $x^4 - x^3 + x^2 - x + 1$

40. $x + 4$

41. $x^3 - x^2 + 1$

42. no

43. yes

44. yes

45. no

46. no

47. yes **48.** yes **49.** yes **50.** no

51. no

52. $x^3 - x^2 + 1$

53. $x^3 - 2x^2 - 2x + 4$, R -35

54. $x^3 - 2x^2 - x + 6$

55. $x^3 - 4x^2 + x$

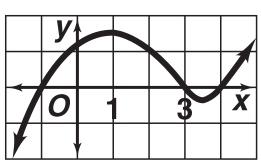
56. **a.** $x + 1$
b. $x^2 + x + 1$
c. $x^3 + x^2 + x + 1$
d. $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

57. **a.** $x^2 - x + 1$
b. $x^4 - x^3 + x^2 - x + 1$
c. $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$
d. $(x + 1)(x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)$

58. By dividing it by a polynomial of degree 1, you are reducing the degree-n polynomial by one, to $n - 1$. The remainder will be constant because it is not divisible by the variable.

59. $x + 2i$

60. Yes; the graph could rise to the right and fall to the left or it could fall to the right and rise to the left.



Answers for Lesson 6-4 Exercises

1. $-2, 1, 5$

2. $-1, 0, 3$

3. $0, 1$

4. $0, 8$

5. $0, -1, -2$

6. $0, -3.5, 1$

7. $0, -0.5, 1.5$

8. $-0.5, 0, 3$

9. $1, 7$

10. 4.8%

11. about $5.78 \text{ ft} \times 6.78 \text{ ft} \times 1.78 \text{ ft}$

12. $(x + 4)(x^2 - 4x + 16)$

13. $(x - 10)(x^2 + 10x + 100)$

14. $(5x - 3)(25x^2 + 15x + 9)$

15. $3, \frac{-3 \pm 3i\sqrt{3}}{2}$

16. $-4, 2 \pm 2i\sqrt{3}$

17. $5, \frac{-5 \pm 5i\sqrt{3}}{2}$

18. $-1, \frac{1 \pm i\sqrt{3}}{2}$

19. $\frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{4}$

20. $-\frac{1}{2}, \frac{1 \pm i\sqrt{3}}{4}$

21. $(x^2 - 7)(x - 1)(x + 1)$

22. $(x^2 + 10)(x^2 - 2)$

23. $(x^2 - 3)(x - 2)(x + 2)$

24. $(x - 2)(x + 2)(x - 1)(x + 1)$

25. $(x - 1)(x + 1)(x^2 + 1)$

26. $2(2x^2 - 1)(x + 1)(x - 1)$

27. $\pm 3, \pm 1$

28. ± 2

29. $\pm 4, \pm 2i$

30. $\pm 3i, \pm\sqrt{2}$

31. $\pm\sqrt{2}, \pm i\sqrt{6}$

32. $\pm i\sqrt{5}, \pm i\sqrt{3}$

33. $3.24, -1, -1.24$

34. $-9, 0$

35. $-3, -2, 1, 2$

36. $1.71, 0.83$

37. $0, 1.54, 8.46$

38. $0, 1.27, 4.73$

39. $-1.04, 0, 6.04$

40. $(n - 1)(n)(n + 1) = 210; 5, 6, 7$

41. D

42. $-\frac{6}{5}, \frac{3 \pm 3i\sqrt{3}}{5}$

43. $\frac{4}{3}, \frac{-2 \pm 2i\sqrt{3}}{3}$

44. $\pm 2\sqrt{2}, \pm 2i\sqrt{2}$

45. $\pm 5, \pm i\sqrt{2}$

46. $\pm 3i, \pm i\sqrt{3}$

47. $0, \pm 2, \pm 1$

48. $\pm\sqrt{10}, \pm i\sqrt{10}$

49. $0, \frac{1}{2} \pm \frac{\sqrt{265}}{10}$

50. $4, -2 \pm 2i\sqrt{3}$

51. $0, 3 \pm \sqrt{3}$

52. $-\frac{3}{2}, 0, 4$

53. $-1, 1, \pm i\sqrt{5}$

54. $-3, -2, 2$

55. $-1, 3, 3$

56. $0, 1, 3$

57. $0, 0, 1, 6$

58. $\pm\sqrt{\frac{3}{2}}, \pm i$

59. $\pm\sqrt{2}, \pm i$

60. Check students' work.

61. $V = x^2(4x - 2)$, 4 in. by 4 in. by 16 in.

62. $x =$ length, $V = x(x - 1)(x - 2)$, 5 meters

63. $-\frac{5}{2}, 1; y = (2x + 5)(x - 1)$

64. $\pm 3, \pm 1; y = (x - 1)(x + 1)(x - 3)(x + 3)$

65. $-1, 2, 2; y = (x + 1)(x - 2)^2$

66. $-2, 1, 3; y = (x + 2)(x - 1)(x - 3)$

67. $-4, -1, 3; y = (x + 4)(x + 1)(x - 3)$

68. A cubic can only have 3 zeros.

69. **a.** Answers may vary. Sample: $x^4 - 9 = 0, \pm\sqrt{3}, \pm i\sqrt{3}$

b. No; two of the roots are imaginary.

70. Answers may vary. Sample: The pink block has volume $a^2(a - 3)$, the orange block has volume $9(a - 3)$, the blue block has volume $3a(a - 3)$, and the purple block has volume 27. Thus $a^3 - 27 = a^2(a - 3) + 3a(a - 3) + 9(a - 3) = (a^2 + 3a + 9)(a - 3)$.

71. **a.** 10

b. 8 and 12

Answers for Lesson 6-5 Exercises

1. $\pm 1, \pm 2; 1$

2. $\pm 1, \pm 2, \pm 3, \pm 6; 1, -2, -3$

3. $\pm 1, \pm 2, \pm 4; -1$

4. $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$; no rational roots

5. $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16; -2$

6. $\pm 1, \pm 3, \pm 5, \pm 15$; no rational roots

7. $2, \pm i\sqrt{5}$

8. $5, \pm i\sqrt{7}$

9. $-3, 1, \frac{7}{2}$

10. $-5, \frac{1 \pm \sqrt{3}}{2}$

11. $\pm\frac{1}{2}, \pm 3$

12. $1, -2, \frac{1 \pm \sqrt{7}}{3}$

13. $-\sqrt{5}, \sqrt{13}$

14. $4 + \sqrt{6}, -\sqrt{3}$

15. $1 + \sqrt{10}, 2 - \sqrt{2}$

16. $1 - i, 5i$

17. $2 - 3i, -6i$

18. $4 + i, 3 - 7i$

19. $x^3 - x^2 + 9x - 9 = 0$

20. $x^3 + 3x^2 - 8x + 10 = 0$

21. $x^3 - 2x^2 + 16x - 32 = 0$

22. $x^3 - 3x^2 - 8x + 30 = 0$

23. $x^3 - 6x^2 + 4x - 24 = 0$

24. $x^3 - x^2 + 2 = 0$

25. $\pm\frac{1}{12}, \pm\frac{1}{6}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{4}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm 3, \pm 6; \frac{1}{2}, \frac{3}{2}, \frac{2}{3}$

26. $\pm\frac{1}{10}, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{1}{2}, \pm\frac{4}{5}, \pm 1, \pm 2, \pm\frac{5}{2}, \pm 4, \pm 5, \pm 10, \pm 20; 2, \frac{2}{5}, \frac{5}{2}$

27. $\pm\frac{7}{3}, \pm\frac{1}{6}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{7}{6}, \pm 1, \pm\frac{3}{2}, \pm 3, \pm\frac{7}{2}, \pm 7, \pm\frac{21}{2}, \pm 21; \frac{1}{3}, -\frac{7}{2}, 1, 3$

28. $\pm\frac{1}{4}, \pm\frac{5}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{5}{8}, \pm\frac{3}{8}, \pm\frac{15}{4}, \pm\frac{5}{2}, \pm 1, \pm\frac{3}{2}, \pm\frac{15}{8}, \pm\frac{15}{2}, \pm 15, \pm 3, \pm 5; -\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

29. $x^4 - 6x^3 + 14x^2 - 24x + 40 = 0$

30. $x^4 - 2x^3 - x^2 + 6x - 6 = 0$

31. $x^4 - 6x^3 + 2x^2 + 30x - 35 = 0$

32. Never true; 5 is not a factor of 8, so by the Rational Root Theorem, 5 is not a root of the equation.

33. Sometimes true; since -2 is a factor of 8, -2 is a possible root of the equation.

34. Always true; use the Rational Root Theorem with $p = a$ and $q = 1$.

35. Sometimes true; since $\sqrt{5}$ and $-\sqrt{5}$ are conjugates, they can be roots of a polynomial equation with integer coefficients.

36. Never true; since $2 + i$ and $-2 - i$ are not conjugates, they cannot be the only imaginary roots of a polynomial equation with integer roots. If their conjugates were also roots, there would be four roots and the equation would have to be of fourth degree.

37. D

38. If $2i$ is a root, then so is $-2i$.

39. Answers may vary. Sample: $x^4 - x^2 - 2 = 0$; roots are $\pm\sqrt{2}$ and $\pm i$.

40. **a.** 2 real, 2 imaginary; 4 imaginary; 4 real

b. 5 real; 3 real, 2 imaginary; 4 imaginary, 1 real

c. Answers may vary. Sample: It has an odd number of real solutions, but it must have at least one real solution.

41. Answers may vary. Sample: You cannot use the Irrational Root Theorem unless the equation has rational coefficients.

42. $x^2 + (-2 + i)x + 12 - 8i = 0$

43. **a–c. Answers may vary. Sample:**

a. $x - 1 - \sqrt{2} = 0$

b. $x^2 - 2(1 + \sqrt{2})x + (1 + \sqrt{2})^2 = 0$

c. -1

Answers for Lesson 6-6 Exercises

1. 3 complex roots; number of real roots: 1 or 3
possible rational roots: ± 1

2. 2 complex roots; number of real roots: 0 or 2
possible rational roots: $\pm\frac{1}{3}, \pm\frac{7}{3}, \pm 1, \pm 7$

3. 4 complex roots; number of real roots: 0, 2, or 4
possible rational roots: 0

4. 5 complex roots; number of real roots: 1, 3, or 5
possible rational roots: $\pm\frac{1}{2}, \pm 1, \pm\frac{5}{2}, \pm 5$

5. 7 complex roots; number of real roots: 1, 3, 5, or 7
possible rational roots: $\pm 1, \pm 3$

6. 1 complex root number of real roots: 1
possible rational roots: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

7. 6 complex roots; number of real roots: 0, 2, 4, or 6
possible rational roots: $\pm\frac{1}{2}, \pm 1, \pm\frac{7}{2}, \pm 7$

8. 10 complex roots; number of real roots: 0, 2, 4, 6, 8, or 10
possible rational roots: ± 1

9. $-1, \frac{1 \pm i\sqrt{7}}{4}$

10. $3, \pm i$

11. $4, \frac{1 \pm i\sqrt{3}}{2}$

12. $2, \pm\sqrt{3}$

13. $\pm 2, \pm\sqrt{2}$

14. $\pm 2, \pm i$

15. $0, \frac{3 \pm 3\sqrt{5}}{2}$

16. $-6, \pm i$

17. 4 complex roots; number of real roots: 0, 2, or 4
possible rational roots: $\pm\frac{1}{2}, \pm 1, \pm 2, \pm\frac{13}{2}, \pm 13, \pm 26$

18. 5 complex roots; number of real roots: 1, 3, or 5
possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

19. 3 complex roots; number of real roots: 1 or 3
possible rational roots: $\pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm\frac{4}{3}, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

20. 6 complex roots; number of real roots: 0, 2, 4, or 6
possible rational roots: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm1, \pm\frac{3}{2}, \pm2, \pm3, \pm4, \pm6,$
$\pm8, \pm12, \pm24$

21. $4, \pm 3i$

22. $-2, \pm\sqrt{5}$

23. $-6, \frac{-1 \pm i}{2}$

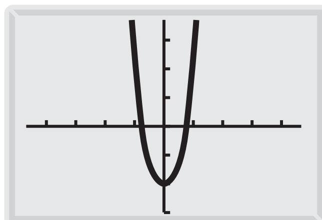
24. $-\frac{1}{4}, -1 \pm 2i$

25. $\frac{1}{2}, \pm 2i\sqrt{5}$

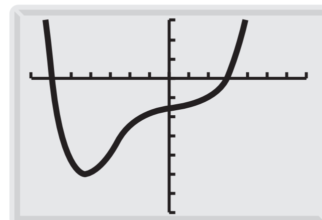
26. $\frac{2}{5}, \frac{-1 \pm i\sqrt{11}}{6}$

27. Answers may vary. Sample: $y = x^4 + 3x^2 + 2$

28. ±0.75



29. $-3.24, 1.24$



30. If you have no constant, then all terms have an x that can be factored out. The resulting expression will have a constant that can be used in the Rational Root Theorem.

31. Yes; for example, $2x^2 - 11x + 5 = 0$ has roots 0.5 and 5.

Answers for Lesson 6-7 Exercises

1. 120

2. 3,628,800

3. 6,227,020,800

4. 720

5. 665,280

6. 120

7. 120

8. 3003

9. **a.** 24
b. 120

10. 8

11. 56

12. 336

13. 1680

14. 6

15. 120

16. 60,480

17. 60

18. 10,897,286,400

19. 4,151,347,200

20. 12

21. 15

22. 56

23. 1

24. 4

25. 35

26. 15

27. 35

28. $\frac{5}{18}$

29. 4368

30. 21

31. 2600

32. 126

33. true because of the Comm. Prop. of Add.

34. true because of the Assoc. Prop. of Mult.

35. False; answers may vary. Sample: $(3 + 2)! = 120$ and $3! + 2! = 8$

36. False; answers may vary. Sample: $(3 \times 2)! = 720$ and $3! \times 2! = 12$

37. False; answers may vary. Sample: $(3!)! = 720$ and $(3!)^2 = 36$

38. False; answers may vary. Sample: $(3!)^2 = 36$ and $3^{(2!)} = 9$

39. C

40. 60

41. 2 ways, because order matters

42. $\frac{4}{19{,}393} \approx .000206$ **43.** 84 **44.** $\frac{12}{49}$

45. 5 **46.** permutation **47.** permutation

48. combination **49.** combination **50.** 330,791,175

51. 210 **52.** 12,650 **53.** 5 **54.** 9504

55. **a.** 56

b. 56

c. Answers may vary. Sample: Each time you choose 3 of the 8 points to use as vertices of a △, the 5 remaining points could be used to form a pentagon.

56. 120 **57.** 3024 **58.** 360 **59.** 24

60. 1680 **61.** 840 **62.** 5040 **63.** 0

64–67. Check students' work.

68. **a.** 2048

b. Answers may vary. Sample: No, because there are too many possible solutions.

69. **a.** The graph for $y = {}_xC_{x-2}$ is identical to the graph for $y = {}_xC_2$ because ${}_2C_{2-2} = {}_2C_2$, ${}_3C_{3-2} = {}_3C_2$, ${}_4C_{4-2} = {}_4C_2$, ${}_5C_{5-2} = {}_5C_2$, etc.

b. Answers may vary. Sample: The function is defined only at discrete whole-number values of x, and not over a smooth range of points as in a continuous function.

70. **a.** 35

b. 6

c. ${}_7C_3 = \frac{7!}{3!\,4!}$, so ${}_7C_3 \cdot 3! = \frac{7!}{4!}$, which is the permutation formula for ${}_7P_3$.

71. **a.** All the terms contain the factors 2 and 5. Since multiplication is commutative, $2 \times 5 = 10$ and 10 times any integer ends in zero.

b. 24 zeros

72. Answers may vary. Sample: 99

73. **a.** 18 people

b. 8568

c. 658,008

d. about 0.013 or 1.3%

Answers for Lesson 6-8 Exercises

1. $a^3 + 3a^2b + 3ab^2 + b^3$
2. $x^2 - 2xy + y^2$
3. $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
4. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$
5. $a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$
6. $x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$
7. $x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$
8. $d^9 + 9d^8 + 36d^7 + 84d^6 + 126d^5 + 126d^4 + 84d^3 + 36d^2 + 9d + 1$
9. $x^3 - 9x^2 + 27x - 27$
10. $a^4 + 12a^3b + 54a^2b^2 + 108ab^3 + 81b^4$
11. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$
12. $x^8 - 32x^7 + 448x^6 - 3584x^5 + 17{,}920x^4 - 57{,}344x^3 + 114{,}688x^2 - 131{,}072x + 65{,}536$
13. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
14. $w^5 + 5w^4 + 10w^3 + 10w^2 + 5w + 1$
15. $s^2 - 2st + t^2$
16. $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$
17. $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
18. $p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 + 35p^3q^4 + 21p^2q^5 + 7pq^6 + q7$
19. $x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$
20. $64 - 48x + 12x^2 - x^3$

21. **a.** about 25%

b. about 21%

c. about 12%

22. about 66%

23. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

24. $x^8 - 40x^7y + 700x^6y^2 - 7000x^5y^3 + 43{,}750x^4y^4 - 175{,}000x^3y^5 + 437{,}500x^2y^6 - 625{,}000xy^7 + 390{,}625y^8$

25. $81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$

26. $x^5 - 20x^4y + 160x^3y^2 - 640x^2y^3 + 1280xy^4 - 1024y^5$

27. $117{,}649 - 201{,}684x + 144{,}060x^2 - 54{,}880x^3 + 11{,}760x^4 - 1344x^5 + 64x^6$

28. $8x^3 + 36x^2y + 54xy^2 + 27y^3$

29. $x^4 + 2x^2y^2 + y^4$

30. $x^6 - 6x^4y + 12x^2y^2 - 8y^3$

31. $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$

32. $x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

33. $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

34. $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

35. $16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$

36. $27x^3 + 135x^2y + 225xy^2 + 125y^3$

37. $64x^6 - 384x^5y + 960x^4y^2 - 1280x^3y^3 + 960x^2y^4 - 384xy^5 + 64y^6$

38. $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$

39. $32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$

40. $2187x^7 + 5103x^6y + 5103x^5y^2 + 2835x^4y^3 + 945x^3y^4 + 189x^2y^5 + 21xy^6 + y^7$

41. $x^6 - 18x^5y + 135x^4y^2 - 540x^3y^3 + 1215x^2y^4 - 1458xy^5 + 729y^6$

42. $x^3 + 15x^2y + 75xy^2 + 125y^3$

43. **a.** about 31%

b. about 16%

c. about 16%

44. **a.** 6

b. 84

45. ${}_8C_4x^4y^4$

46. 9

47. 7, $7r^6s$

48. $594x^{10}$

49. $80x^2$

50. $27x^8$

51. $264x^{10}$

52. x^{11}

53. $64y^6$

54. $-823{,}680x^8y^7$

55. $314{,}928x^7$

56. $29{,}568x^{10}y^6$

57. $1716x^{12}y^{14}$

58. Answers may vary. Sample: Since one of the terms is negative and it is alternately raised to odd and even powers, the term is negative when raised to an odd power and positive when raised to an even power.

59. **a.** $(s + 0.5)^3$

b. $s^3 + 1.5s^2 + 0.75s + 0.125$

60. The exponent of q should be 5 because the exponent of q should be the degree (7) minus the exponent of p.

61. $13, d^{12}, 12d^{11}e$

62. $16, x^{15}, -15x^{14}y$

63. $6, 32a^5, 80a^4b$

64. $8, x^7, -21x^6y$

65. **a.** -4

b. $(1 - i)^4 = 1 - 4i - 6 + 4i + 1 = -4$ ✓

66. $(-1 + \sqrt{3} \cdot i)^3 = -1 + 3i\sqrt{3} + 9 - 3i\sqrt{3} = 8$ ✓

67. Answers may vary. Sample: A coin is tossed five times with the probability of heads on each toss 0.5. Write an expression for the probability of exactly 2 heads being tossed.

68. **a.** $(k + 1)! = (k + 1) \cdot (k) \cdot (k - 1) \cdot \ldots \cdot 1 = (k + 1)[(k) \cdot (k - 1) \cdot \ldots \cdot 1] = (k + 1) \cdot k!$

b. The derivation below finds a common denominator for the fractions that represent ${}_nC_k$ and ${}_nC_{k+1}$, and then uses algebra to show that ${}_nC_k + {}_nC_{k+1} = {}_{n+1}C_{k+1}$. In addition, the identity from part (a) is used three times.

$${}_nC_k + {}_nC_{k+1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!} =$$

$$\frac{(k+1)n!}{(k+1)!(n-k)!} + \frac{(n-k)n!}{(k+1)!(n-k)!} =$$

$$\frac{(k+1+n-k)n!}{(k+1)!(n-k)!} = \frac{(n+1)n!}{(k+1)!(n-k)!} =$$

$$\frac{(n+1)!}{(k+1)!(n-k)!} = \frac{(n+1)!}{(k+1)!((n+1)-(k+1))!} = {}_{n+1}C_{k+1}$$

c. If you consider the row of Pascal's Triangle containing just 1 to be row zero, ${}_4C_2$ is 6, the third entry in the fourth row. ${}_4C_3$ is 4, the fourth entry in the fourth row. ${}_5C_3$ is 10, the fourth entry in the fifth row.
${}_4C_2 + {}_4C_3 = 6 + 4 = 10 = {}_5C_3$