HIGH SCHOOL MATHEMATICS CONTESTS
Math League Press, P.O. Box 17, Tenafly, New Jersey 07670-0017

Contest Number 2 | ***Any calculator is always allowed.*** Answers must be exact ***or*** have 4 (or more) significant digits, correctly rounded. | **December 5, 1995**

Name ____________ **Teacher** ____________ **Grade Level** _____ **Score** ____

Time Limit: 30 minutes | *Answer Column*

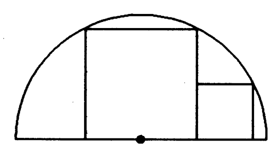
2-1. When the first 1995 positive odd primes are multiplied together, what is the units' digit of the product?

2-1.

2-2. When $x = 10$, the expression $\sqrt{1+2+3+x}$ has the value 4. What are all four integers $x < 10$ for which $\sqrt{1+2+3+x}$ has an integral value?

2-2.

2-3. Two squares are inscribed in a semicircle as shown. If the area of the smaller square is 25, what is the area of the larger square?



2-3.

2-4. The College of Hard Knox belongs to a six-school league in which each school plays four games with each of the other schools. No tied games ever occur, and the other five schools finished this season having won, respectively, 20%, 30%, 35%, 60%, and 80% of the league games they played. What was the The College of Hard Knox's final winning record in the league this season (expressed as a percent)?

2-4.

2-5. Place one *non-zero* digit in each box below so the resulting equation is true:

$\square\square$% of $\square\square\square$ = 400.

2-5. Put your answer in the shaded boxes at the left.

2-6. Both x and y are positive numbers less than 2. Every positive number less than 2 is equally likely to be the value of x; and every positive number less than 2 is equally likely to be the value of y. What is the probability that x and y differ by less than 1?

2-6.

Contest # 2 — *Answers & Solutions* — 12/5/95

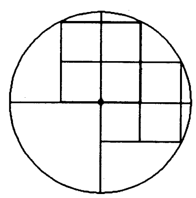
Problem 2-1

Every odd multiple of 5 ends in $\boxed{5}$.

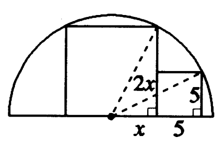
Problem 2-2

By definition, $\sqrt{1+2+3+x} \geq 0$. Since $x < 10$, it follows that $0 \leq \sqrt{6+x} < 4$. The answers are the solutions to $\sqrt{6+x} = 0$, $\sqrt{6+x} = 1$, $\sqrt{6+x} = 2$, and $\sqrt{6+x} = 3$. These answers are $\boxed{-6, -5, -2, 3}$.

Problem 2-3



Method I: Divide the larger square into 4 congruent smaller squares. Rotate the larger square 90° clockwise, and it is clear that the smaller square is one-quarter of the larger square. The larger square's area is $4 \times 25 = \boxed{100}$.



Method II: In the diagram, each dotted segment is both the hypotenuse of a right triangle and a radius of the circle; so the dotted segments are congruent. In one right triangle, the legs are x and $2x$. In the other, the legs are 5 and $x+5$. By the Pythagorean Theorem, $x^2+(2x)^2 = 5^2+(x+5)^2$. Solving $x = 5$; so a side of the large square is 10, and the area of the large square is 100.

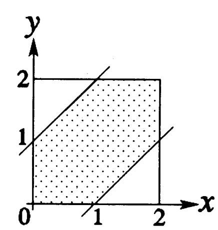
Problem 2-4

At the season's end, each team, on average, has won 50% of its games; so the sum of all six winning percents is always 6(50%) = 300%. The sum for the five schools whose records were known was 225%, so Hard Knox's winning record was 300% − 225% = $\boxed{75\%}$.

Problem 2-5

Let the unknown 2-digit number be a and the unknown 3-digit number be b. Since a% of b = 400, it follows that $ab = 40\,000 = 2^6 \times 5^4$. Neither a nor b contains the digit 0, so neither has a factor of 10 and neither can have both 2 and 5 as factors. Thus, $a = 2^6 = 64$, $b = 5^4 = 625$, and $400 = \boxed{64\% \text{ of } 625}$.

Problem 2-6



Since $0 < x < 2$ and $0 < y < 2$, we can use the coordinate plane to model the given conditions. The graph of the model is a 2×2 square in the first quadrant. Every point (x,y) inside the square meets the requirements that x and y are both positive and less than 2. The values of x and y differ by less than 1 everywhere in the shaded region, since $|x - y| < 1$ if and only if $y > x - 1$ and $y < x + 1$. The unshaded region of the 2×2 square can be reassembled to form a 1×1 square, so its area is 1. The required probability is the fractional part of the square that is shaded. This probability, which is the area of the shaded region divided by the area of the square, equals $\boxed{\frac{3}{4}}$.